

Curs 4

2015/2016

# Dispozitive și circuite de microunde pentru radiocomunicații

# Disciplina 2015/2016

- 2C/1L, **DCMR (CDM)**
- **Minim 7 prezente (curs+laborator)**
- Curs - **sl. Radu Damian**
  - Marti 18-20, P2
  - E – 60% din nota
  - probleme + (2p prez. curs)
    - 3p=+0.5p
  - toate materialele permise
- Laborator – **sl. Radu Damian**
  - Miercuri 8-14 impar (14.10.2015 – prez. obligatorie)
  - L – 25% din nota
  - P – 15% din nota

# Fotografii

Start Didactic Master Colectiv Cercetare Studenti Admin

Note Lista Studenti Fotografii Statistici

Grupa 5403

Nr. Student	Prezent	Nr. Student	Prezent	Nr. Student	Prezent
1 ANGHIELUS IONUT-MARIUS	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	2 ANTIGHIN FLORIN-RAZVAN	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	3 ANTONICA BIANCA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:
4 APOSTOL PAVEL-MANUEL	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	5 BALASCA IULIAN-PETRU	<input checked="" type="checkbox"/> Puncte: 0 Nota: 0 Obs:	6 BOSTAN ANDREI-PETRICIA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:
7 BOTEZAT EMANUEL	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	8 BUTUNOI GEORGE-MADALIN	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	9 CHILEA SALUCA-MARIA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:
10 CHERITOIU ECATERINA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	11 COJOC MARIUS	<input checked="" type="checkbox"/> Puncte: 0 Nota: 0 Obs:	12 COJOCARI AURA-FLORINA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:

Nr. Student	Prezent
2 ANTIGHIN FLORIN-RAZVAN	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:

# Reprezentare logaritmică

$$\text{dB} = 10 \cdot \log_{10} (P_2 / P_1)$$

$$\text{dBm} = 10 \cdot \log_{10} (P / 1 \text{ mW})$$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 $\mu$ W
-30 dBm	= 1 $\mu$ W
-60 dBm	= 1 nW

$$[\text{dBm}] + [\text{dB}] = [\text{dBm}]$$

$$[\text{dBm/Hz}] + [\text{dB}] = [\text{dBm/Hz}]$$

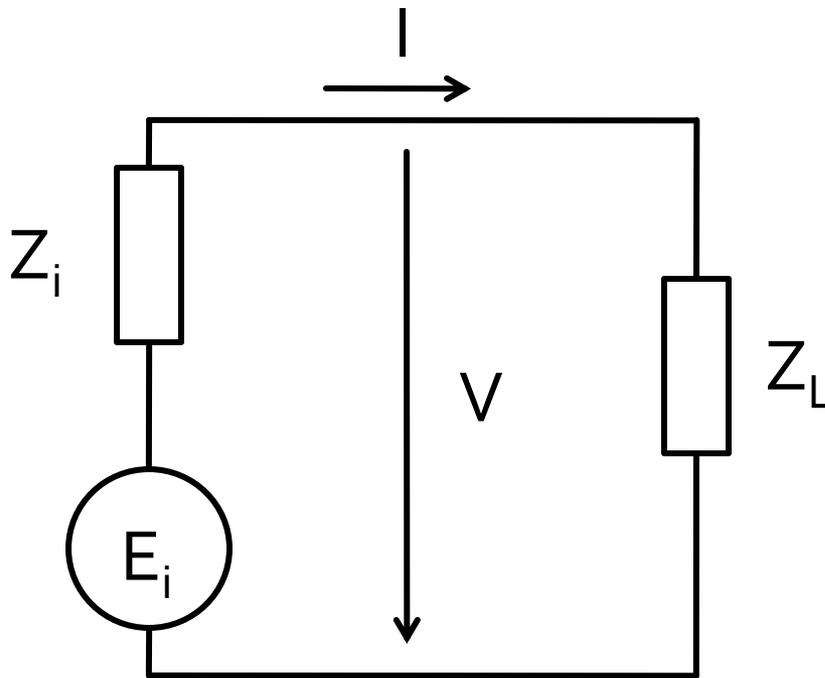
$$[\text{x}] + [\text{dB}] = [\text{x}]$$

# Recapitulare

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# Adaptare

- Generator adaptat la sarcina ?



- valori impedanta ?
- reflexii ?

# Adaptare dpdv al puterii

$$R_i > 0, R_L > 0 \quad P_L = \frac{|E_i|^2}{4R_i + \frac{(R_i - R_L)^2}{R_L} + \frac{(X_i + X_L)^2}{R_L}}$$

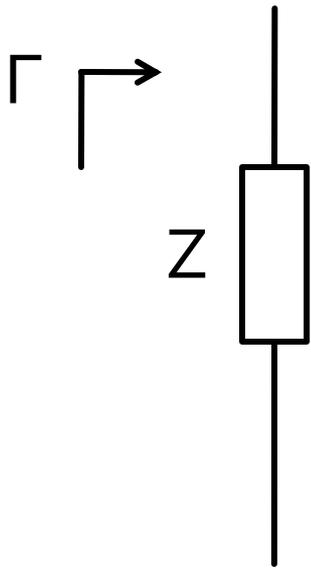
$$P_{L\max} = \frac{|E_i|^2}{4R_i} \equiv P_a \quad R_L = R_i, X_L = -X_i$$

- Puterea disponibilă (available)

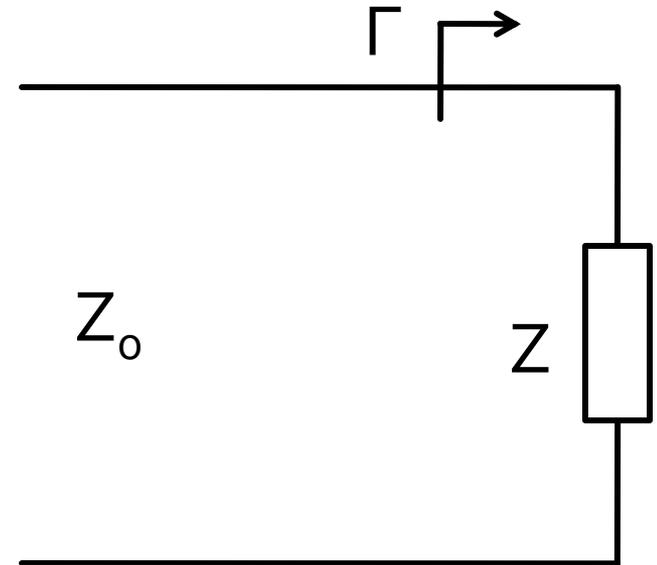
$$Z_L = Z_i^*$$

# Coeficient de reflexie

- Un  $Z_0$  oarecare ales ca referinta



$$\Gamma = \frac{Z - Z_0^*}{Z + Z_0}$$



# Adaptare dpdv al puterii

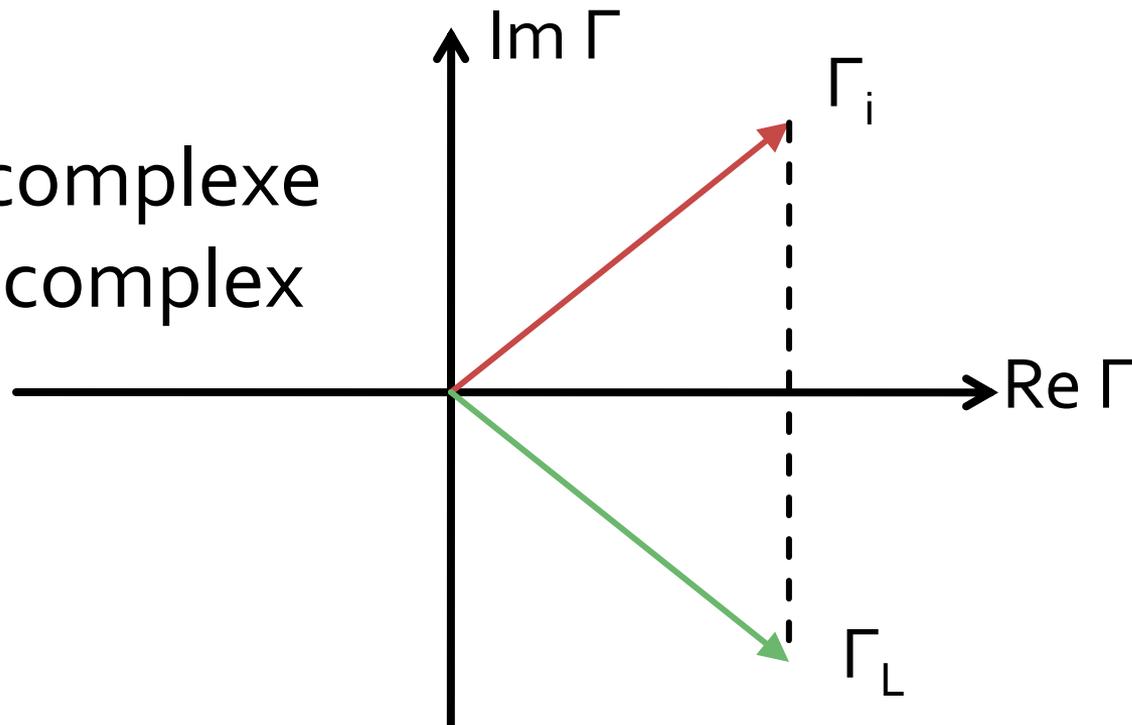
$$Z_L = Z_i^*$$

Daca se alege un  $Z_0$  real

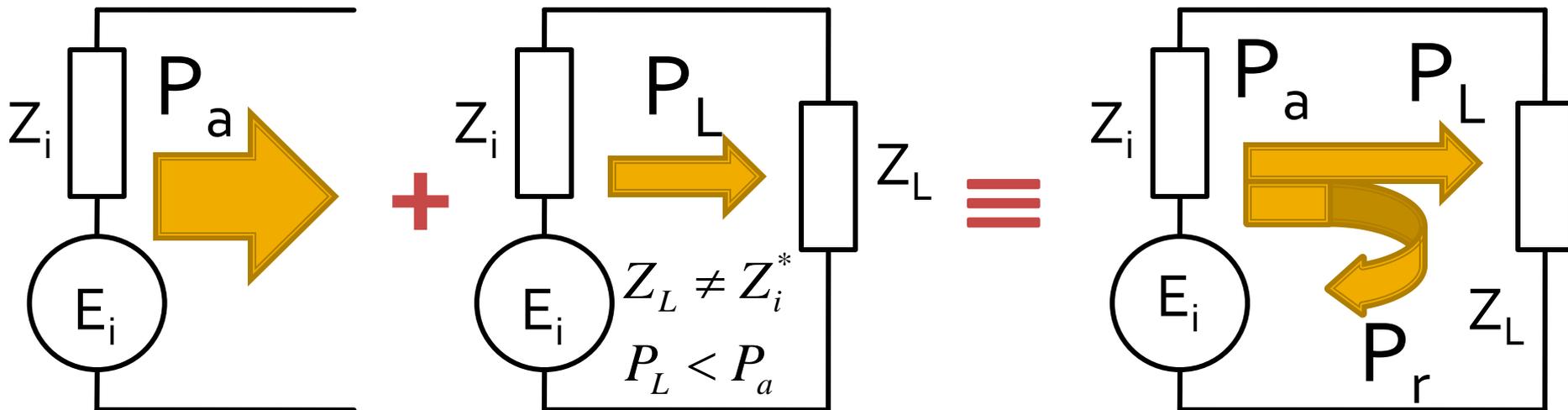
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- numere complexe
- in planul complex

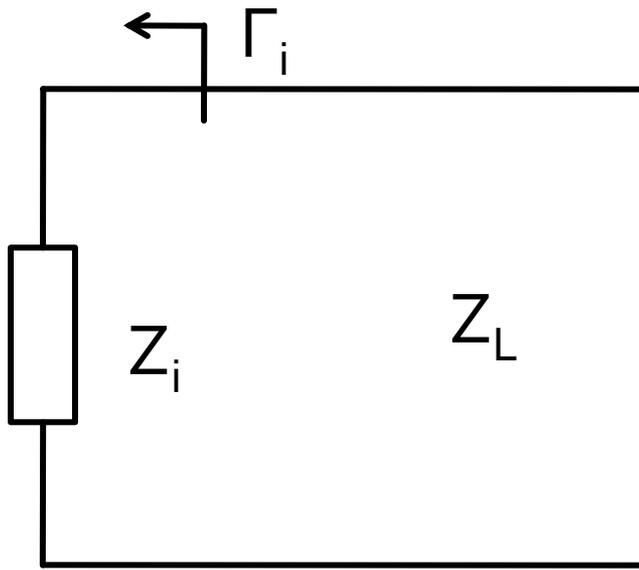


# Reflexie de putere / Model

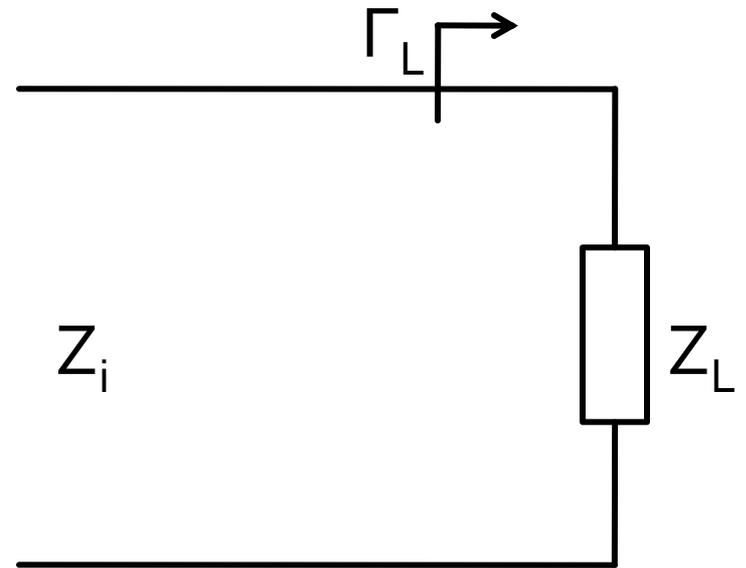


- Generatorul are posibilitatea de a oferi o anumita putere maxima de semnal  $P_a$
- Pentru o sarcina oarecare, acestuia i se ofera o putere de semnal mai mica  $P_L < P_a$
- Se intampla "ca si cum" (model) o parte din putere se reflecta  $P_r = P_a - P_L$
- Puterea este o marime **scalara!**

# Coeficienti de reflexie



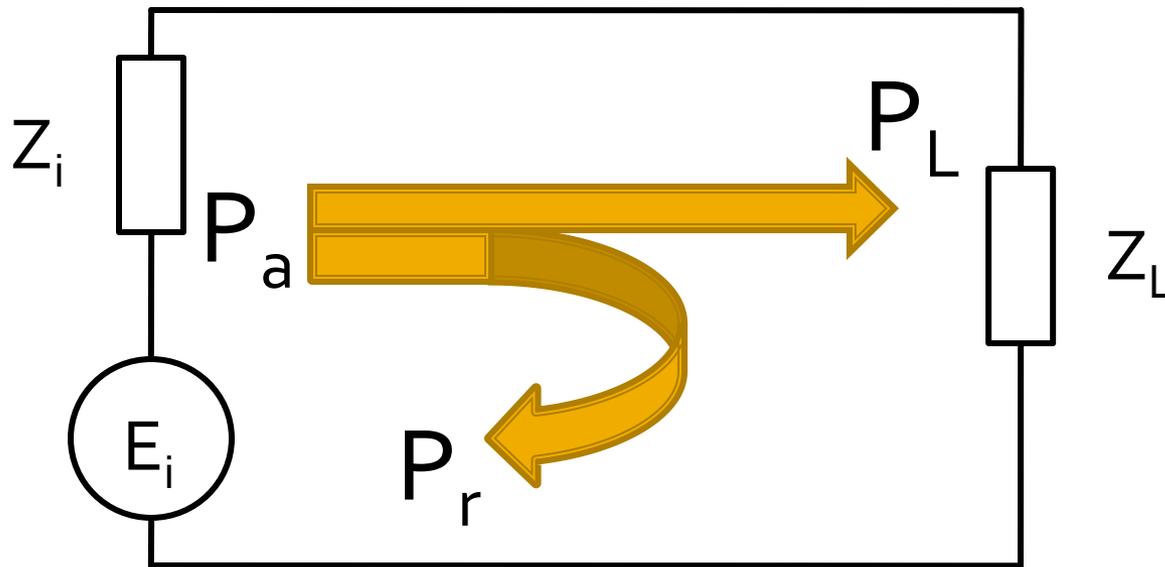
$$\Gamma_i = \frac{Z_i - Z_L^*}{Z_i + Z_L}$$



$$\Gamma_L = \frac{Z_L - Z_i^*}{Z_L + Z_i}$$

$$|\Gamma_i| = |\Gamma_L| \equiv |\Gamma|$$

# Reflexie de putere / Model



$$P_a = \frac{|E_i|^2}{4R_i}$$

$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

$$P_r = P_a - P_L = \frac{|E_i|^2}{4R_i} \cdot \left[ \frac{(R_i - R_L)^2 + (X_i + X_L)^2}{(R_i + R_L)^2 + (X_i + X_L)^2} \right] = P_a \cdot |\Gamma|^2$$

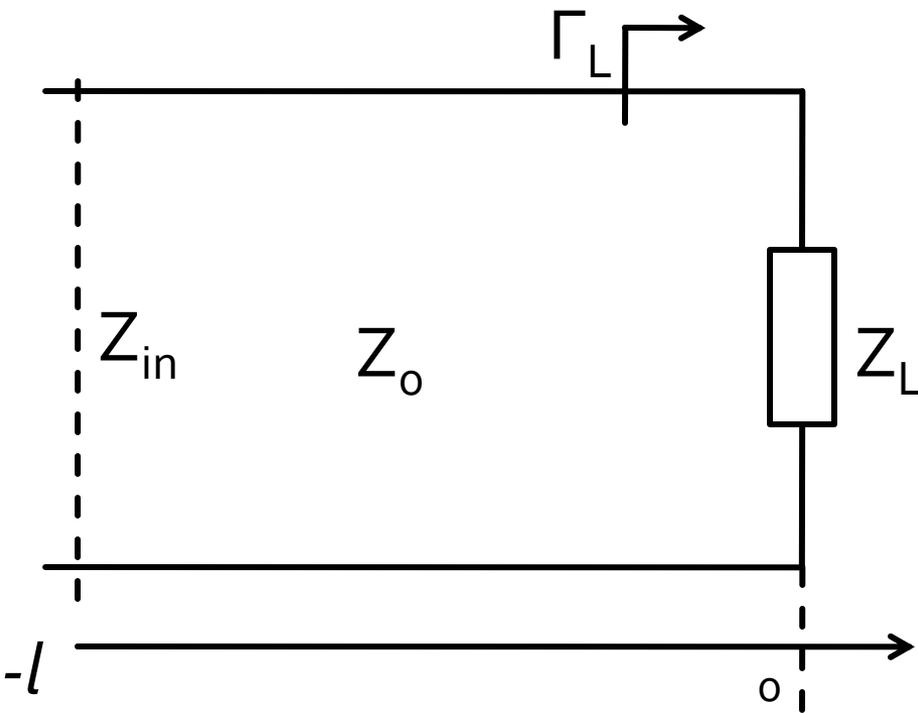
■ coeficient de reflexie in putere

$$P_L = P_a - P_r = P_a - P_a \cdot |\Gamma|^2 = P_a \cdot (1 - |\Gamma|^2)$$

# Linii de transmisie in mod TEM

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# Linie fara pierderi



$$V(-l) = V_0^+ e^{j\beta \cdot l} + V_0^- e^{-j\beta \cdot l}$$

$$I(-l) = \frac{V_0^+}{Z_0} e^{j\beta \cdot l} - \frac{V_0^-}{Z_0} e^{-j\beta \cdot l}$$

$$Z_{in} = \frac{V(-l)}{I(-l)} \quad Z_{in} = Z_0 \cdot \frac{1 + \Gamma \cdot e^{-2j\beta \cdot l}}{1 - \Gamma \cdot e^{-2j\beta \cdot l}}$$

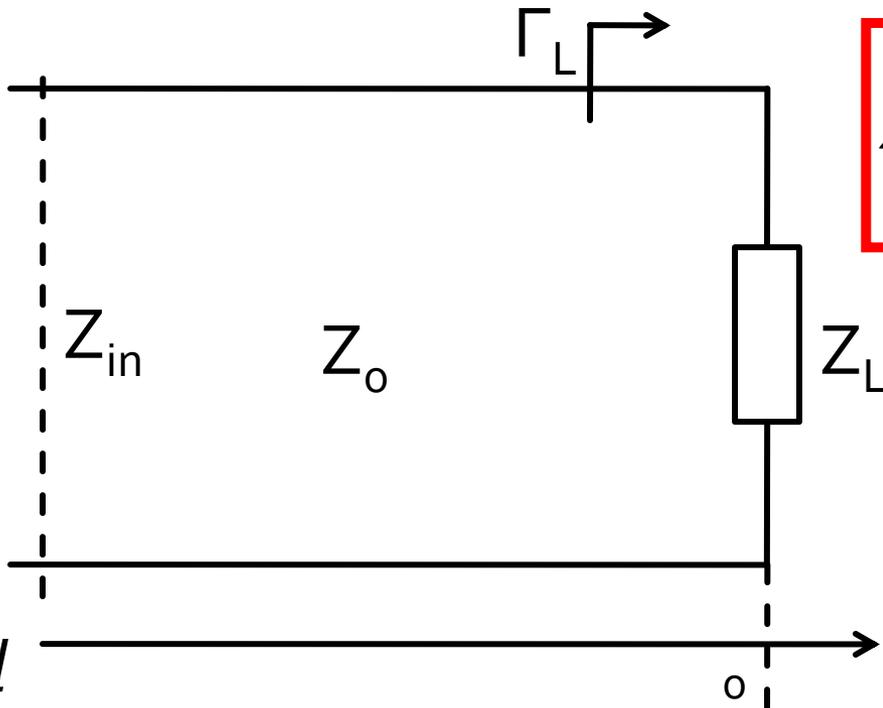
- impedanta la intrarea liniei

$$Z_{in} = Z_0 \cdot \frac{(Z_L + Z_0) \cdot e^{j\beta \cdot l} + (Z_L - Z_0) \cdot e^{-j\beta \cdot l}}{(Z_L + Z_0) \cdot e^{j\beta \cdot l} - (Z_L - Z_0) \cdot e^{-j\beta \cdot l}}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

# Linie fara pierderi

- impedanta la intrarea liniei de impedanta caracteristica  $Z_0$ , de lungime  $l$ , terminata cu impedanta  $Z_L$

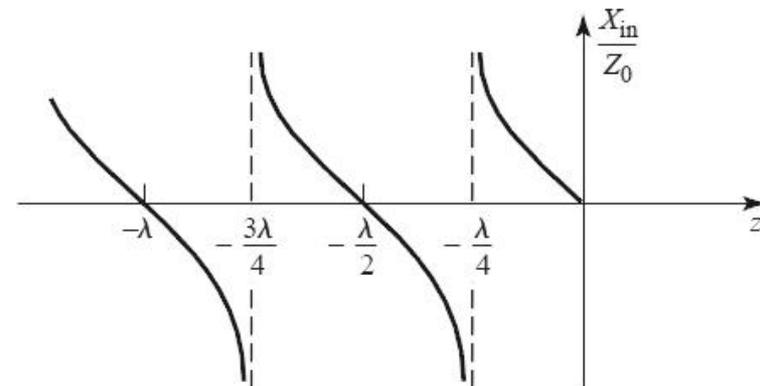
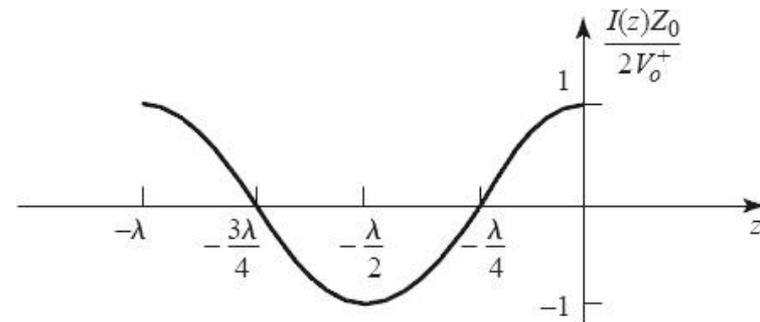
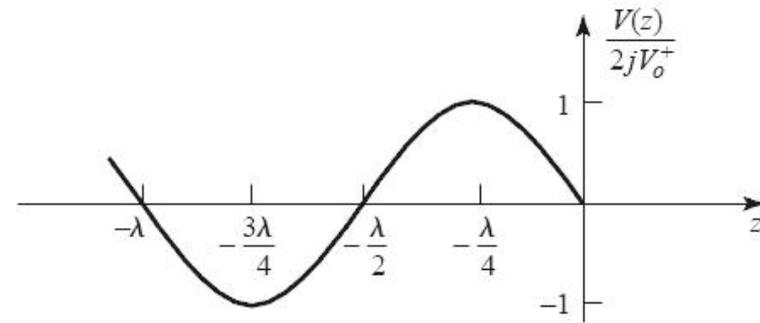


$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

# Linie in scurtcircuit

- reactanța pură
  - +/- → in funcție de  $l$

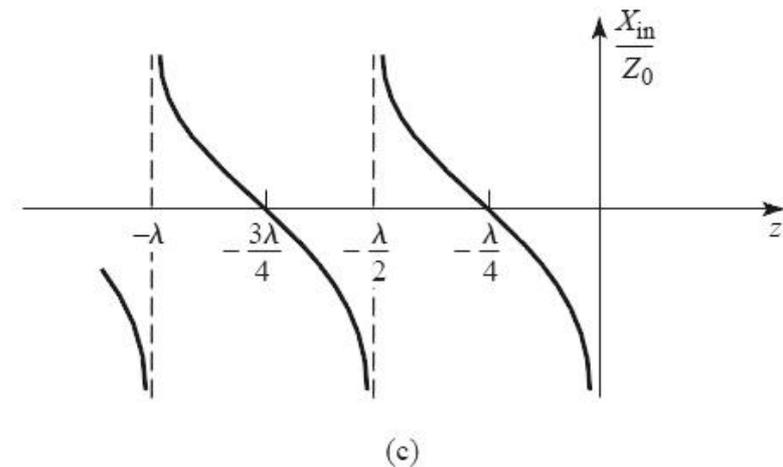
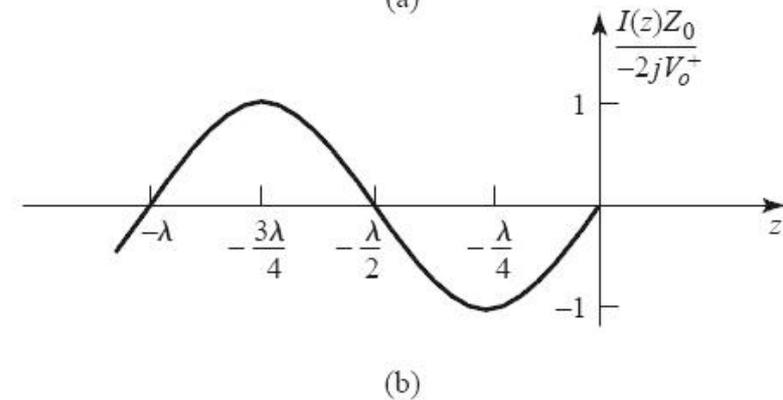
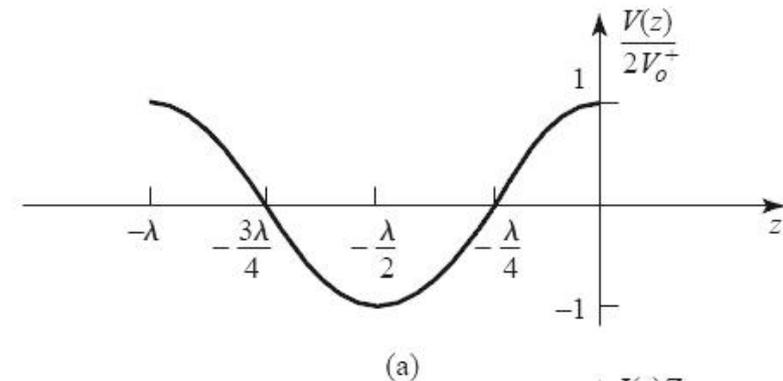
$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$



# Linie in gol

- reactanța pură
  - +/- → in funcție de  $l$

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$



**Continuare**

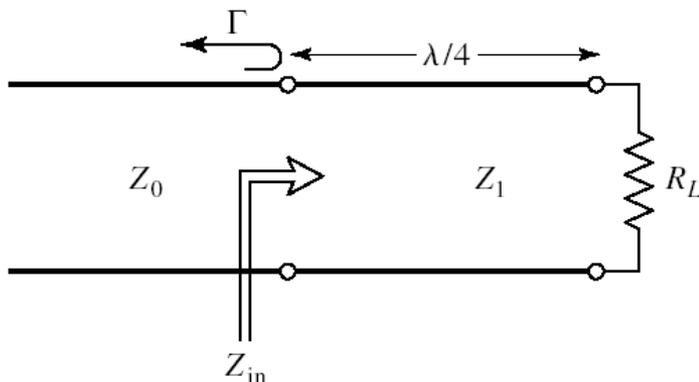
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# Adaptarea de impedanță

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# Transformatorul in sfert de lungime de unda

- Feed line – linie de intrare cu impedanta caracteristica  $Z_0$
- Sarcina cu impedanta  $R_L$
- Dorim adaptarea sarcinei la fider cu o linie de lungime  $\lambda/4$  si impedanta caracteristica  $Z_1$

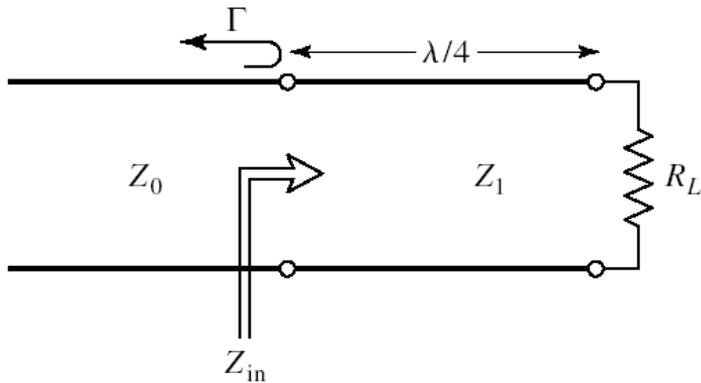


$$Z_{in} = Z_1 \frac{1 + \Gamma e^{-2j\beta l}}{1 - \Gamma e^{-2j\beta l}}$$

$$\Gamma_o = \frac{V_0^-}{V_0^+} = \frac{R_L - Z_1}{R_L + Z_1}$$

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$

# Transformatorul in sfert de lungime de unda



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{in} = \frac{Z_1^2}{R_L}$$

$$\Gamma_{in} = \frac{Z_1^2 - Z_0 \cdot R_L}{Z_1^2 + Z_0 \cdot R_L} \quad \Gamma_{in} = 0 \quad Z_1 = \sqrt{Z_0 R_L}$$

- Pe fider ( $Z_0$ ) avem doar unda progresiva
- Pe linia in sfert de lungime de unda ( $Z_1$ ) avem unda stationara

# Transformatorul in sfert de lungime de unda

## ■ Punct de vedere fizic

$$\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 + T_1 T_2 \Gamma_2 \Gamma_3^2 - T_1 T_2 \Gamma_2^2 \Gamma_3^3 + \dots$$

$$= \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n.$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

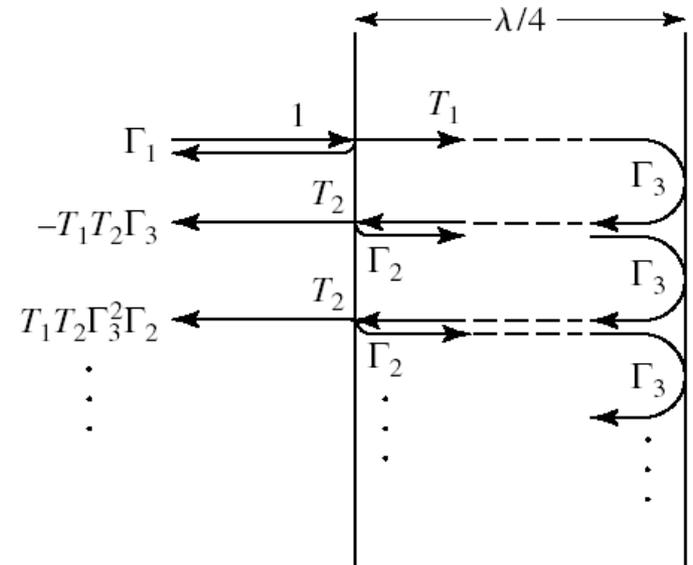
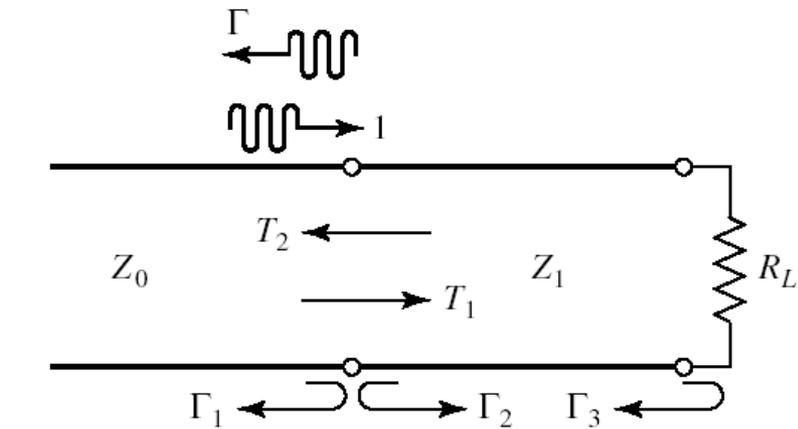
$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1,$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1},$$

$$T_1 = \frac{2Z_1}{Z_1 + Z_0},$$

$$T_2 = \frac{2Z_0}{Z_1 + Z_0}.$$

$$\left. \begin{array}{l} T_1 \\ T_2 \end{array} \right\} T = 1 - \Gamma$$



# Transformatorul in sfert de lungime de unda

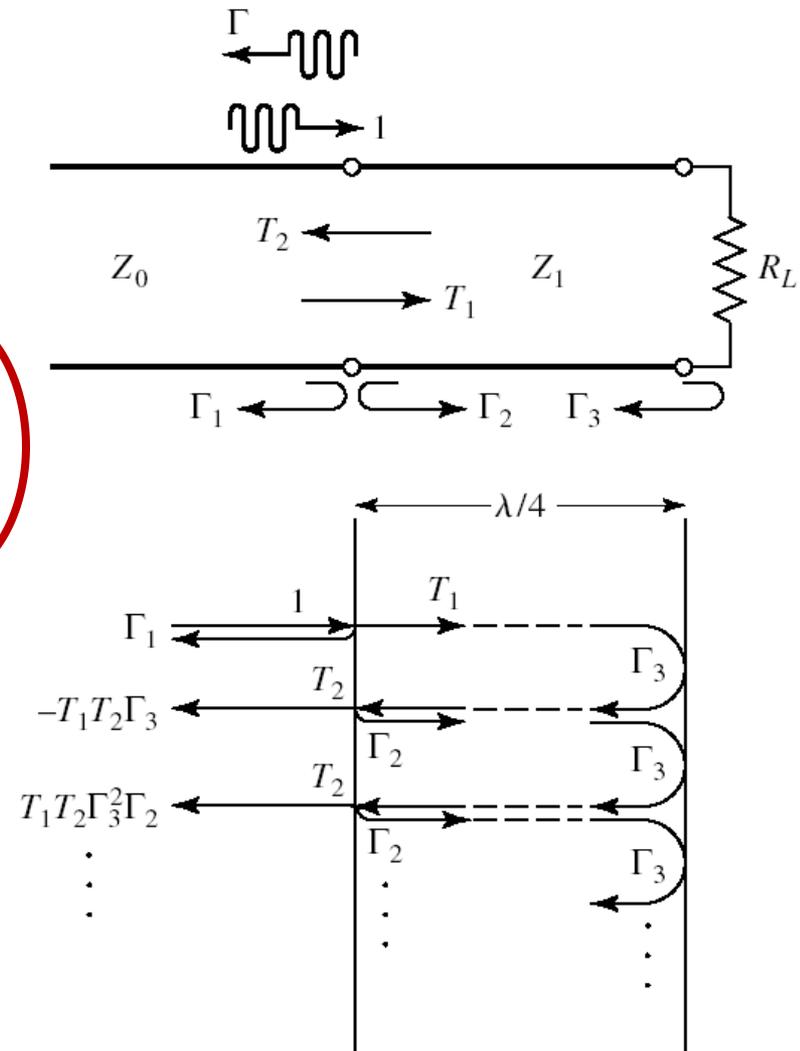
## ■ Punct de vedere fizic

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad \text{for } |x| < 1,$$

$$\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3}.$$

$$\Gamma_1 - \Gamma_3(\Gamma_1^2 + T_1 T_2) = \frac{2(Z_1^2 - Z_0 R_L)}{(Z_1 + Z_0)(R_L + Z_1)},$$

$$\Gamma = 0 \Leftrightarrow Z_1^2 - Z_0 \cdot R_L = 0$$



# Característica de frecuencia

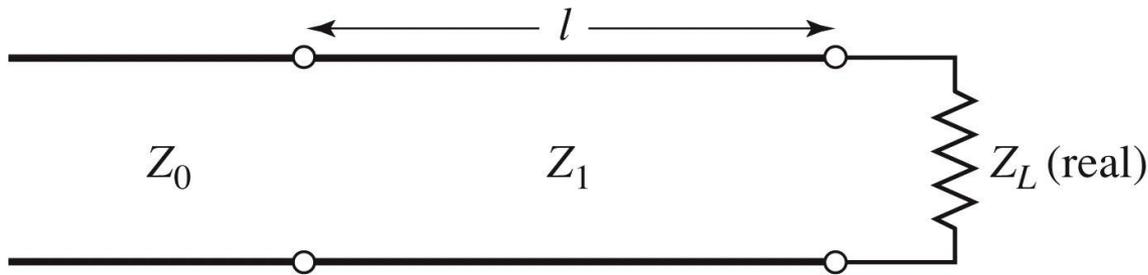


Figure 5.10  
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$$Z_1 = \sqrt{Z_0 \cdot Z_L}$$

- (doar) la frecventa  $f_0$

$$l = \frac{\lambda_0}{4} \quad \beta_0 \cdot l = \frac{2\pi}{\lambda_0} \cdot \frac{\lambda_0}{4} = \frac{\pi}{2}$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta \cdot l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

$$\theta = \beta \cdot l$$

$$t = \tan(\beta \cdot l)$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot t}{Z_1 + j \cdot Z_L \cdot t}$$

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_1(Z_L - Z_0) + jt(Z_1^2 - Z_0Z_L)}{Z_1(Z_L + Z_0) + jt(Z_1^2 + Z_0Z_L)}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0Z_L}}$$

# Caracteristica de frecventa

- calitatea adaptarii  $\equiv$  coeficient de reflexie in putere

$$\begin{aligned} |\Gamma| &= \frac{|Z_L - Z_0|}{[(Z_L + Z_0)^2 + 4t^2 Z_0 Z_L]^{1/2}} \\ &= \frac{1}{\{(Z_L + Z_0)^2 / (Z_L - Z_0)^2 + [4t^2 Z_0 Z_L / (Z_L - Z_0)^2]\}^{1/2}} \\ &= \frac{1}{\{1 + [4Z_0 Z_L / (Z_L - Z_0)^2] + [4Z_0 Z_L t^2 / (Z_L - Z_0)^2]\}^{1/2}} \\ &= \frac{1}{\{1 + [4Z_0 Z_L / (Z_L - Z_0)^2] \sec^2 \theta\}^{1/2}}, \end{aligned}$$

$$\sec \theta = \frac{1}{\cos \theta} \rightarrow$$

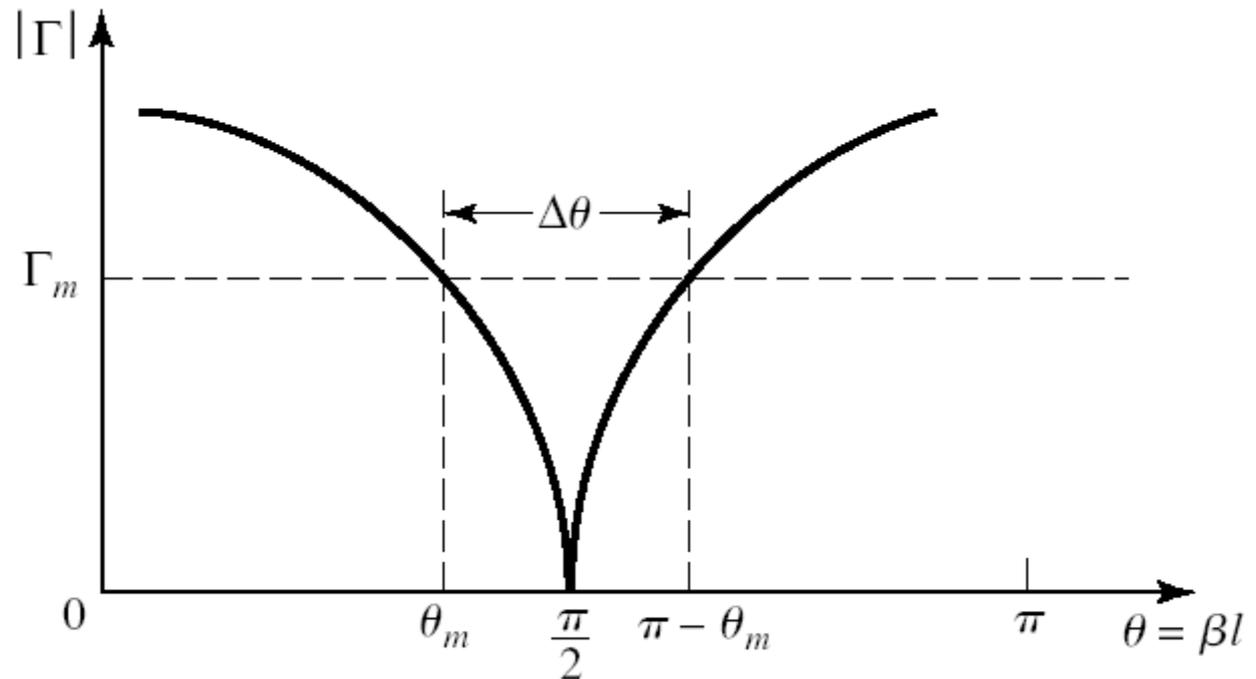
$$\sec^2 \theta = 1 + \tan^2 \theta = 1 + t^2$$

# Caracteristica de frecventa

- ne intereseaza frecventa in jurul frecventei la care facem adaptarea (banda ingusta)

$$f \approx f_0 \quad l \approx \frac{\lambda_0}{4} \quad \theta \approx \frac{\pi}{2} \quad \sec^2 \theta = 1 + \tan^2 \theta \gg 1$$

$$|\Gamma| \cong \frac{|Z_L - Z_0|}{2 \cdot \sqrt{Z_0 \cdot Z_L}} \cdot |\cos \theta|$$



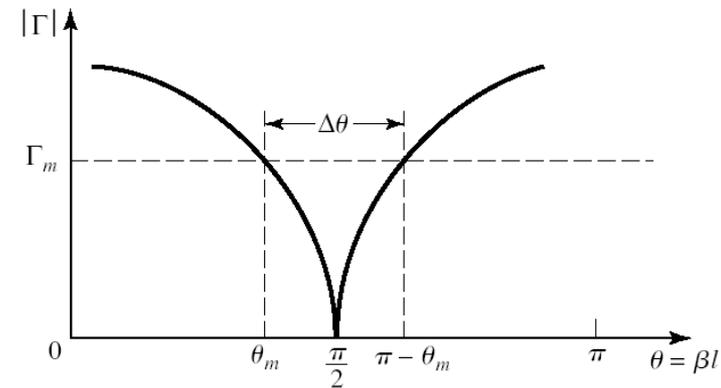
# Caracteristica de frecventa

- Definim un maxim acceptat pentru coeficientul de reflexie  $\Gamma_m$  care va defini banda adaptarii,  $\theta_m$

$$\frac{1}{\Gamma_m^2} = 1 + \left( \frac{2\sqrt{Z_0 Z_L}}{Z_L - Z_0} \sec \theta_m \right)^2,$$

$$\cos \theta_m = \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}.$$

$$\Delta\theta = 2 \left( \frac{\pi}{2} - \theta_m \right)$$



- in linii TEM

$$\theta = \beta \cdot l = \beta \cdot \frac{\lambda_0}{4} = \frac{2\pi \cdot f}{v_f} \cdot \frac{1}{4} \cdot \frac{v_f}{f_0} = \frac{\pi \cdot f}{2f_0}$$

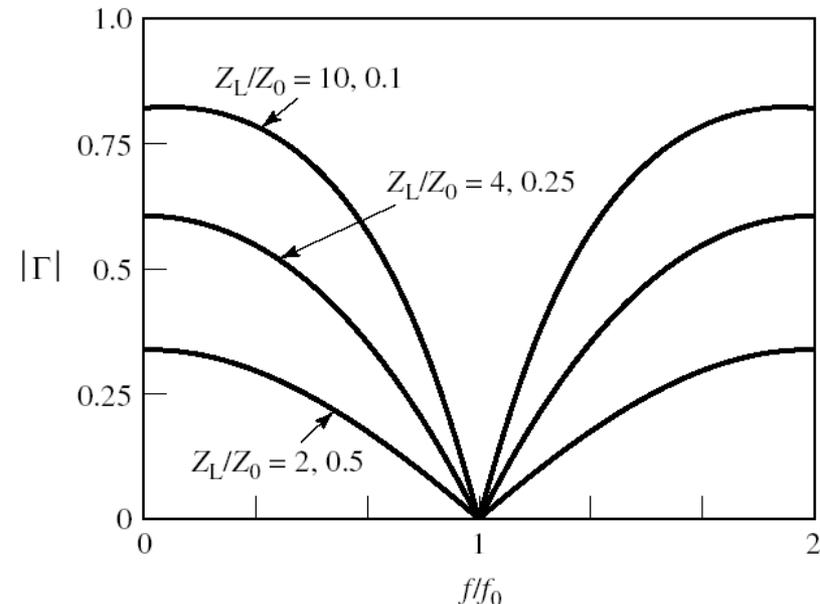
$$f_m = \frac{2 \cdot \theta_m \cdot f_0}{\pi}$$

$$\frac{\Delta f}{f_0} = \frac{2 \cdot (f_0 - f_m)}{f_0} = 2 - \frac{4 \cdot \theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1} \left[ \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \cdot \frac{2\sqrt{Z_0 \cdot Z_L}}{|Z_L - Z_0|} \right]$$

# Caracteristica de frecventa

- Pentru linii non TEM constanta de propagare nu depinde linear de frecventa, dar in practica influenta este minora in banda ingusta
- Sunt neglijate reactantele introduse de discontinuitati ( $Z_0 \rightarrow Z_1$ ). Compensarea se face printr-o mica modificare a lungimii liniei
- Banda depinde de dezadaptarea initiala

cu cat dezadaptarea este mai mica  
cu atat banda se obtine mai larga



# Exemplu

- Transformator de adaptare cu o singura sectiune ( $\lambda/4$ ) pentru a adapta o sarcina de  $10\Omega$  la o linie de  $50\Omega$  la frecventa  $f_0=3\text{GHz}$ 
  - banda pentru  $\text{SWR}<1.5$

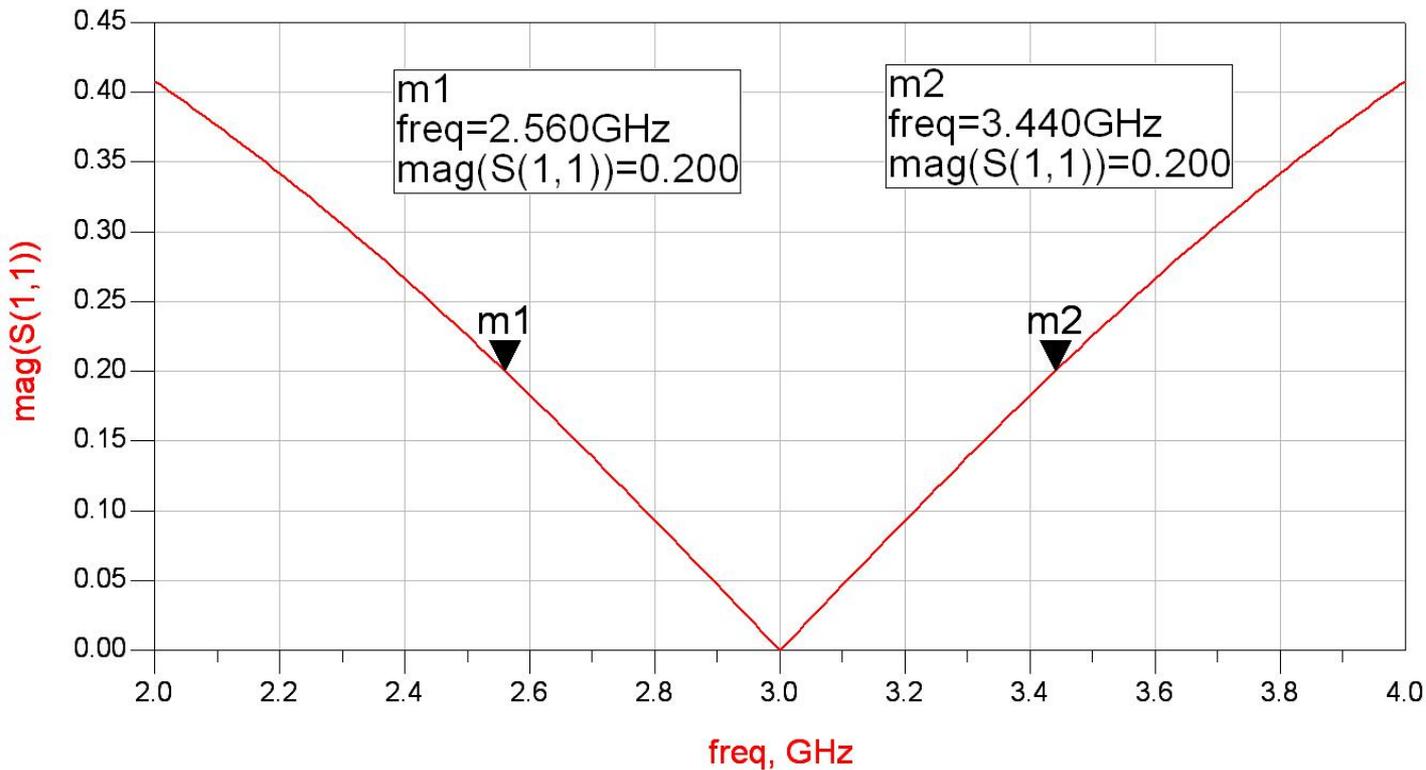
$$Z_1 = \sqrt{Z_0 Z_L} = \sqrt{(50)(10)} = 22.36 \Omega,$$

$$\Gamma_m = \frac{\text{SWR} - 1}{\text{SWR} + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2.$$

$$\begin{aligned} \frac{\Delta f}{f_0} &= 2 - \frac{4}{\pi} \cos^{-1} \left[ \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|} \right] \\ &= 2 - \frac{4}{\pi} \cos^{-1} \left[ \frac{0.2}{\sqrt{1 - (0.2)^2}} \frac{2\sqrt{(50)(10)}}{|10 - 50|} \right] \\ &= 0.29, \text{ or } 29\%. \end{aligned}$$

# Simulare

## ■ simulare ADS

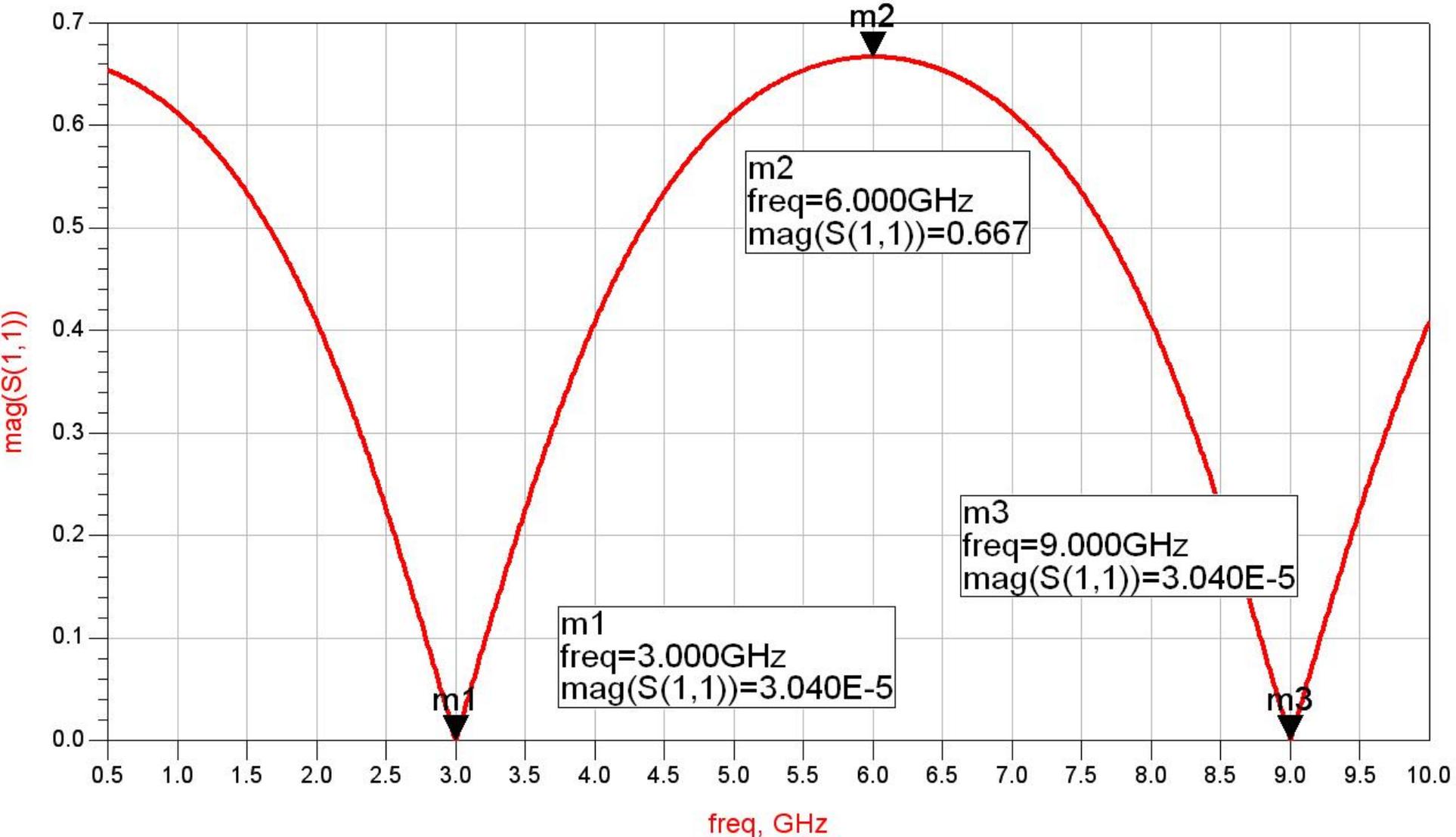


$$\Delta f = 0.88 \text{GHz}$$

$$|\Gamma(3\text{GHz})| = 3 \cdot 10^{-5}$$

$$\frac{\Delta f}{f_0} = \frac{0.88}{3} = 0.2933$$

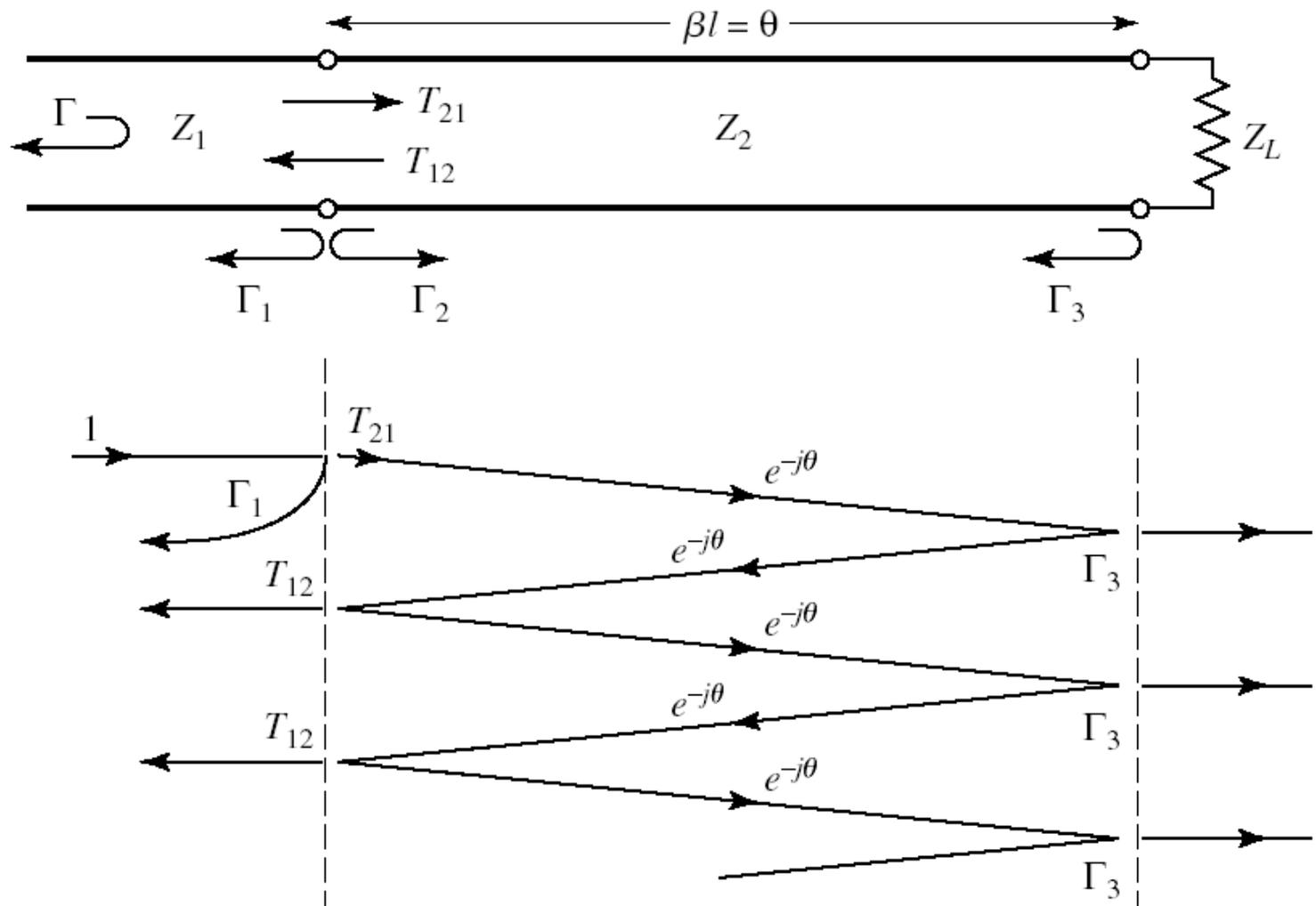
# Simulare banda larga



# Transformatoare de impedanta multisectiune

- Transformatorul in sfert de lungime de unda permite adaptarea oricarei impedante reale cu orice impedanta a fiderului (liniei).
- Daca banda necesara este mai mare decat cea oferita de transformatorul in sfert de lungime de unda se folosesc transformatoare multisectiune
  - caracteristica binomiala
  - tip Cebîșev

# Teoria reflexiilor mici



# Teoria reflexiilor mici

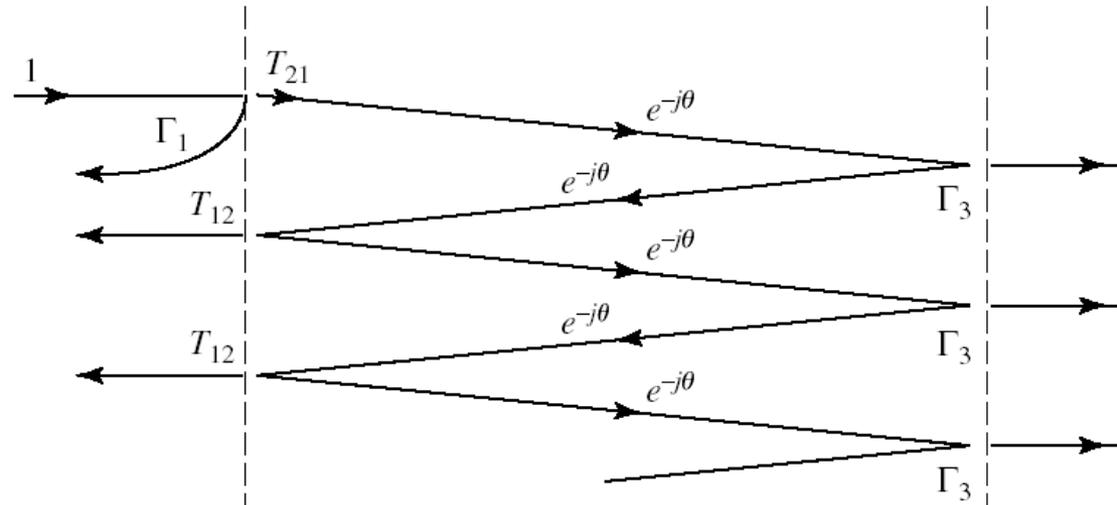
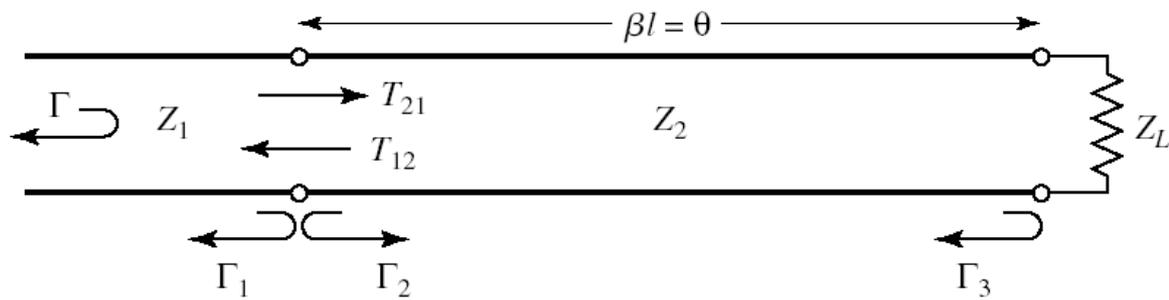
$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\Gamma_2 = -\Gamma_1$$

$$\Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2}$$

$$T_{21} = 1 + \Gamma_1 = \frac{2 \cdot Z_2}{Z_1 + Z_2}$$

$$T_{12} = 1 + \Gamma_2 = \frac{2 \cdot Z_1}{Z_1 + Z_2}$$



$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^2 \cdot \Gamma_2 \cdot e^{-4j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^3 \cdot \Gamma_2^2 \cdot e^{-6j\theta} + \dots$$

$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

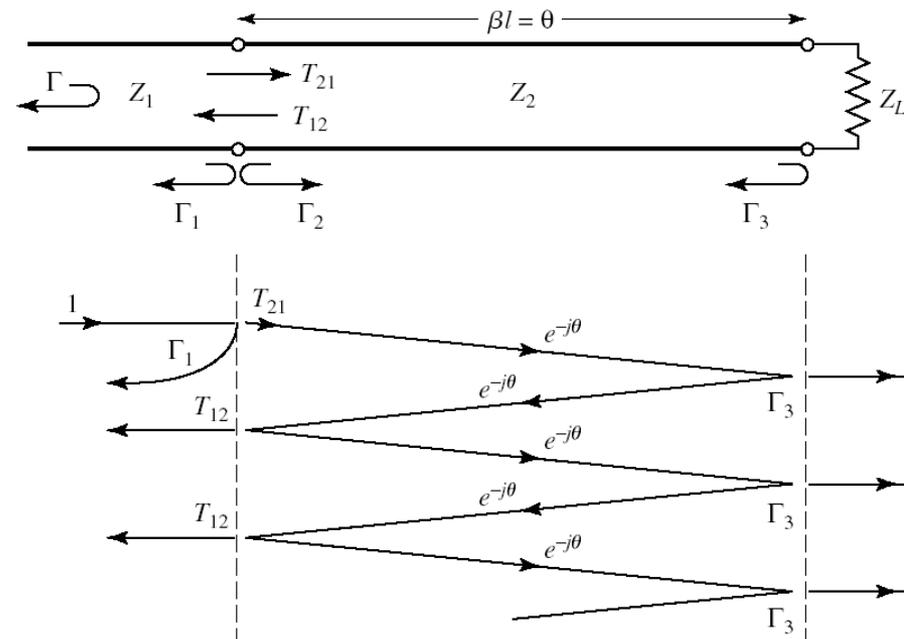
# Teoria reflexiilor mici

$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

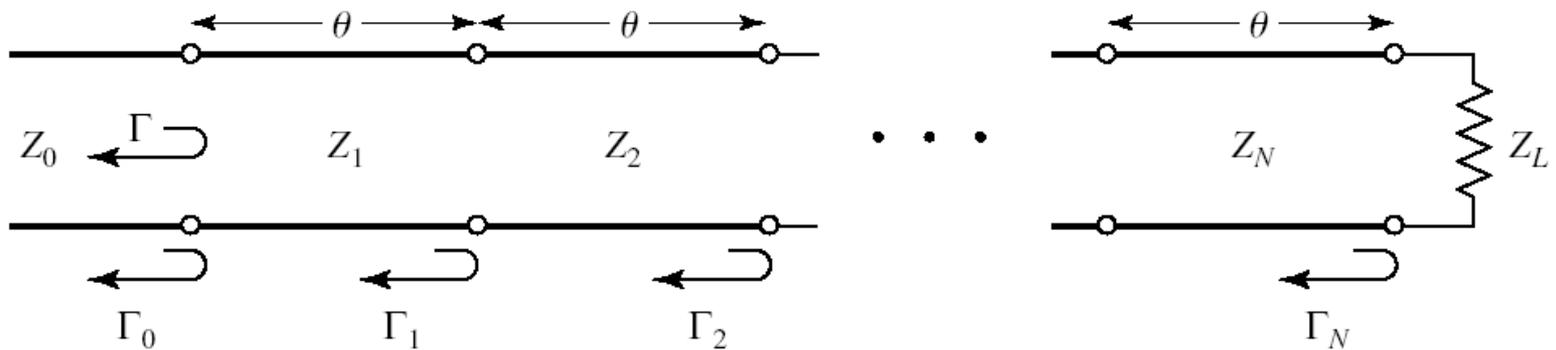
$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1$$

$$\Gamma = \frac{\Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}}{1 + \Gamma_1 \cdot \Gamma_3 \cdot e^{-2j\theta}}$$

$$\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}$$



# Transformatoare cu mai multe sectiuni



- Presupunem ca toate impedantele

**cresc sau descrec uniform**

- Toti coeficientii de reflexie vor fi reali si de acelasi semn

- Anterior  $\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta} \Rightarrow$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$n = \overline{1, N-1}$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

# Transformatoare cu mai multe sectiuni

- Realizez transformatorul **simetric**
- Aceasta **nu** implica faptul ca impedantele sunt egale

$$\Gamma_0 = \Gamma_N, \Gamma_1 = \Gamma_{N-1}, \Gamma_2 = \Gamma_{N-2} \dots$$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma(\theta) = e^{-jN\theta} \cdot [\Gamma_0 \cdot (e^{jN\theta} + e^{-jN\theta}) + \Gamma_1 \cdot (e^{j(N-2)\theta} + e^{-j(N-2)\theta}) + \Gamma_2 \cdot (e^{j(N-4)\theta} + e^{-j(N-4)\theta}) + \dots]$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots + \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

ultimul termen:

$$\dots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$$

$$\dots \Gamma_{(N-1)/2} \cdot \cos \theta \quad n \text{ impar}$$

# Transformatoare cu mai multe sectiuni

- Coeficient de reflexie

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \dots + a_N \cdot x^N$$

- aleg coeficientii astfel incat sa obtin o variatie dorita (a polinomului)

# Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- Raspunsul acestui transformator este de tip maxim plat in jurul frecventei de adaptare
- Pentru N sectiuni se anuleaza primele N-1 derivate ale functiei  $|\Gamma(\theta)|$

$$f(x) = A \cdot (1+x)^N$$

$$\Gamma(\theta) = A \cdot (1 + e^{-2j\theta})^N$$

$$|\Gamma(\theta)| = |A| \cdot |e^{-j\theta}|^N \cdot |e^{j\theta} + e^{-j\theta}|^N = 2^N \cdot |A| \cdot |\cos \theta|^N$$

$$\left| \Gamma\left(\frac{\pi}{2}\right) \right| = 0; \quad \frac{d^n}{d\theta^n} |\Gamma(\theta)| \Big|_{\theta=\frac{\pi}{2}} = 0 \quad n = \overline{1, N-1} \quad l = \frac{\lambda}{4} \Rightarrow \theta = \beta \cdot l = \frac{\pi}{2}$$

# Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- $A, \theta \rightarrow 0$ , liniile de lungime 0, dispar

$$\Gamma(0) = 2^N \cdot A = \frac{Z_L - Z_0}{Z_L + Z_0} \quad A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0}$$

- dezvoltarea binomului

$$f(x) = (1+x)^N = C_N^0 + C_N^1 \cdot x + \dots + C_N^n \cdot x^n + \dots + C_N^N \cdot x^N$$

$$C_N^n = \frac{N!}{(N-n)!n!}$$

- Coeficientii de reflexie

$$\Gamma(\theta) = A \cdot (1 + e^{-2j\theta})^N \quad \Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_n = A \cdot C_N^n$$

# Transformatoare cu mai multe sectiuni cu caracteristica binomiala

## ■ Proiectare

$$\Gamma_n = A \cdot C_N^n$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \cong \frac{1}{2} \ln \frac{Z_{n+1}}{Z_n} \qquad \ln x \cong 2 \cdot \frac{x-1}{x+1} \quad x \cong 1$$

$$\ln \frac{Z_{n+1}}{Z_n} \cong 2 \cdot \Gamma_n = 2 \cdot A \cdot C_N^n = 2 \cdot 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0} \cong 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

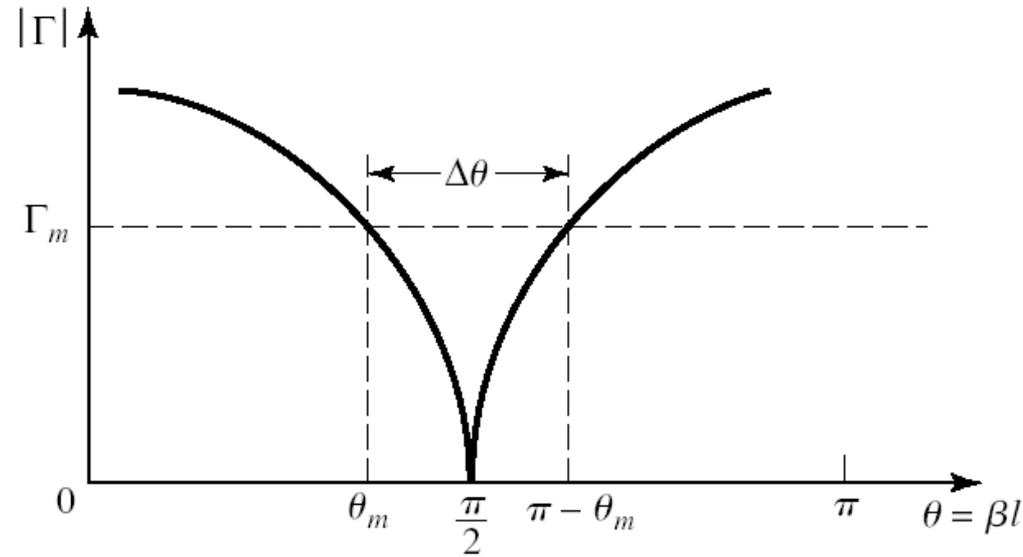
$$\ln Z_{n+1} \cong \ln Z_n + 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

# Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- Banda,  $\Gamma_m$  maxim tolerat

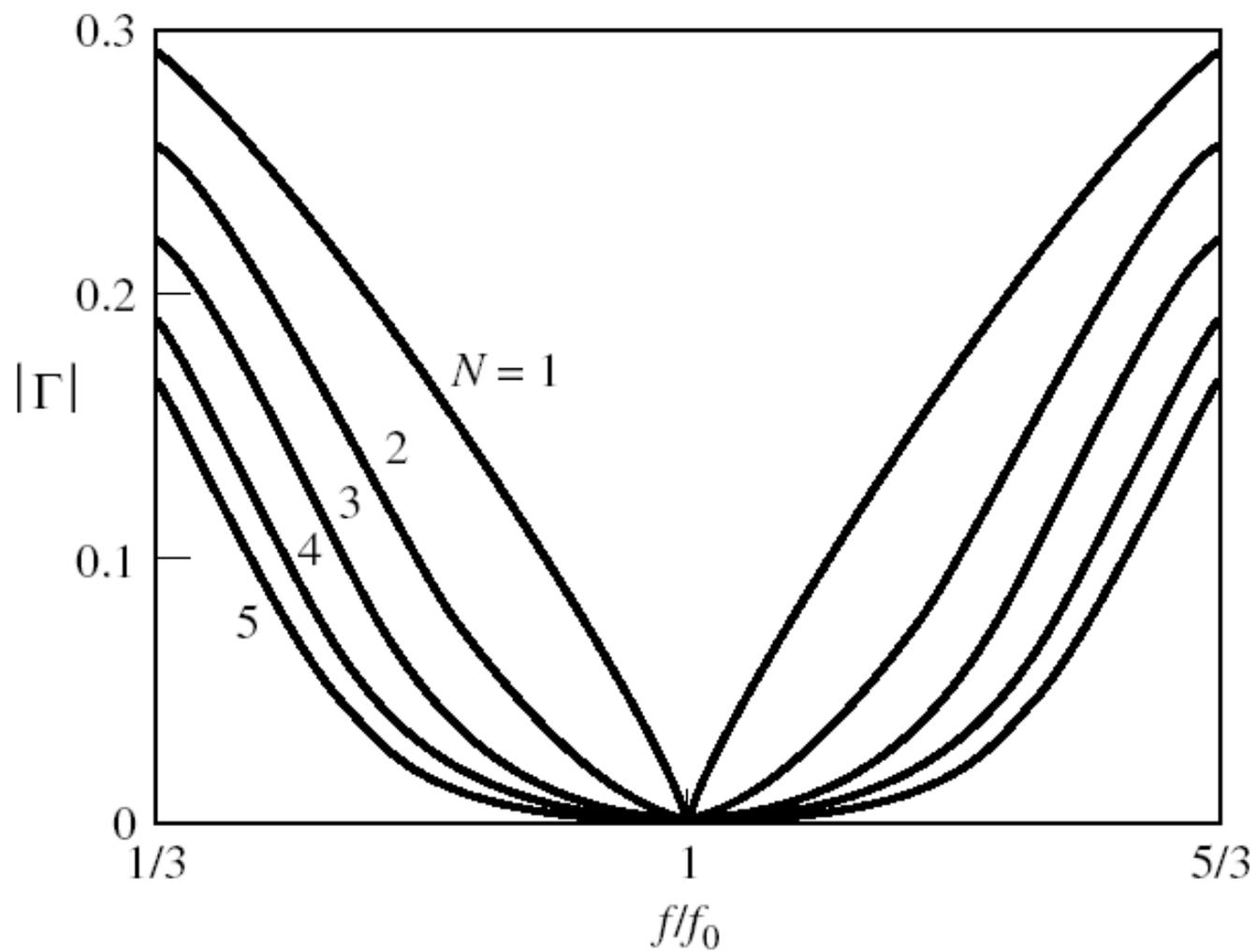
$$\Gamma_m = |\Gamma(\theta_m)| = 2^N \cdot |A| \cdot |\cos \theta_m|^N$$

$$\theta_m = \cos^{-1} \left[ \frac{1}{2} \left( \frac{\Gamma_m}{|A|} \right)^{\frac{1}{N}} \right]$$



$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1} \left[ \frac{1}{2} \left( \frac{\Gamma_m}{|A|} \right)^{\frac{1}{N}} \right]$$

# Banda



# Transformatoare cu mai multe sectiuni cu caracteristica binomiala rezultate exacte

$Z_L/Z_0$	$N = 2$		$N = 3$			$N = 4$			
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

$Z_L/Z_0$	$N = 5$					$N = 6$					
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$	$Z_5/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$	$Z_5/Z_0$	$Z_6/Z_0$
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

# Exemplu

- Transformator de adaptare cu 3 sectiuni pentru a adapta o sarcina de  $30\Omega$  la o linie de  $100\Omega$  la frecventa  $f_0=3\text{GHz}$ ,  $\Gamma_m=0.1$ 
  - $N = 3$

$$Z_L = 30\Omega \quad Z_0 = 100\Omega$$

$$A = 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0} \approx \frac{1}{2^{N+1}} \ln \frac{Z_L}{Z_0} = -0.07525$$

$$C_3^0 = \frac{3!}{3! \cdot 0!} = 1 \quad C_3^1 = \frac{3!}{2! \cdot 1!} = 3 \quad C_3^2 = \frac{3!}{1! \cdot 2!} = 3$$

# Exemplu

$n = 0$

$$\ln Z_1 = \ln Z_0 + 2^{-N} C_3^0 \ln \frac{Z_L}{Z_0} = \ln 100 + 2^{-3} \cdot 1 \cdot \ln \frac{30}{100} = 4.455$$

$$Z_1 = 86.03\Omega$$

$n = 1$

$$\ln Z_2 = \ln Z_1 + 2^{-N} C_3^1 \ln \frac{Z_L}{Z_0} = \ln 86.03 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 4.003$$

$$Z_2 = 54.77\Omega$$

$n = 2$

$$\ln Z_3 = \ln Z_2 + 2^{-N} C_3^2 \ln \frac{Z_L}{Z_0} = \ln 54.77 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 3.552$$

$$Z_3 = 34.87\Omega$$

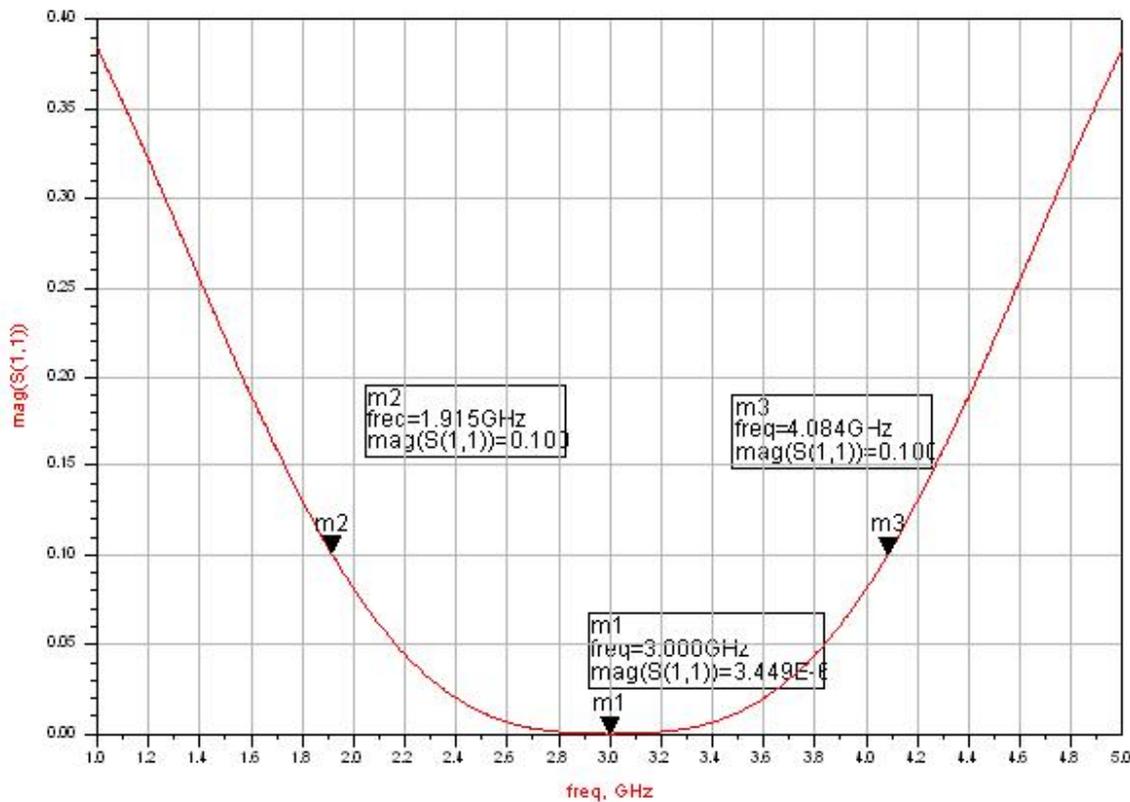
# Exemplu

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \arccos \left[ \frac{1}{2} \left( \frac{\Gamma_m}{|A|} \right)^{1/N} \right] = 2 - \frac{4}{\pi} \arccos \left[ \frac{1}{2} \left( \frac{0.1}{0.07525} \right)^{1/3} \right] = 0.74$$

$$\Delta f = 2.22 \text{GHz}$$

# Simulare

## ■ Similar Lab. 1



$$\Delta f = 2.169 \text{GHz}$$

$$|\Gamma(3\text{GHz})| = 3.5 \cdot 10^{-6}$$

# Transformatoare cu mai multe sectiuni de tip Cebîșev

- Raspunsul acestui transformator este de tip echiriplu in jurul frecventei de adaptare
- mareste banda in detrimentul riplului in banda de adaptare
- Se egaleaza functia  $\Gamma(\theta)$  cu un polinom Cebîșev

# Polinoame Cebîşev

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

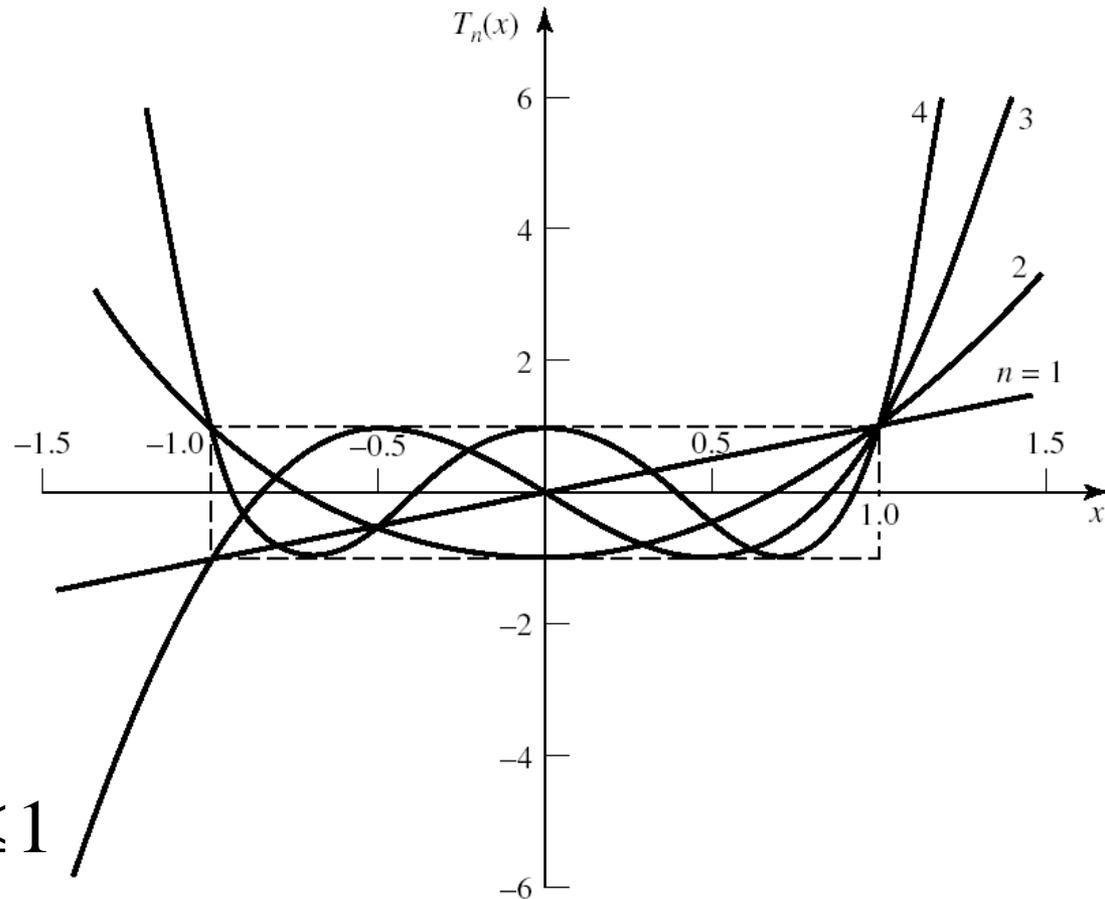
$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

## ■ Echiriplu

$$-1 \leq x \leq 1 \Rightarrow |T_n(x)| \leq 1$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$



# Polinoame Cebîşev

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \dots + a_N \cdot x^N$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots + \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

ultimul termen:  $\dots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$

$$\dots \Gamma_{(N-1)/2} \cdot \cos \theta \quad n \text{ impar}$$

$$x = \cos \theta \quad T_n(\cos \theta) = \cos(n\theta)$$

$$T_n(x) = \cos(n \arccos(x)) \quad |x| < 1 \quad T_n(x) = \cosh(n \cosh^{-1}(x)) \quad |x| > 1$$

# Transformatoare cu mai multe sectiuni de tip Cebîșev

- Schimbare de variabila

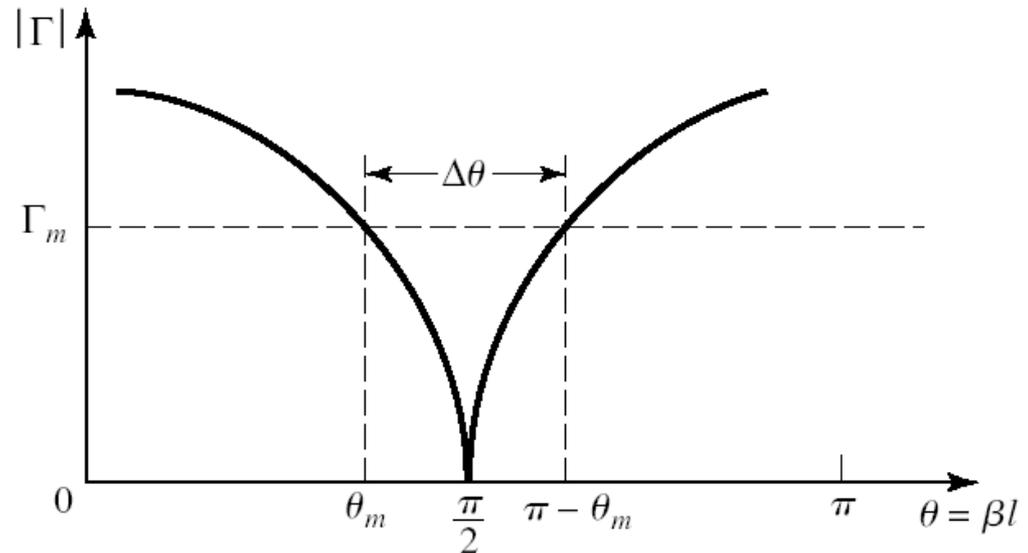
$$\theta = \theta_m \rightarrow x = 1$$

$$\theta = \pi - \theta_m \rightarrow x = -1$$

$$x \equiv \frac{\cos \theta}{\cos \theta_m}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$x = \sec \theta_m \cos \theta$$



# Transformatoare cu mai multe sectiuni de tip Cebîșev

$$T_1(\sec \theta_m \cos \theta) = \sec \theta_m \cos \theta$$

$$T_2(\sec \theta_m \cos \theta) = \sec^2 \theta_m (1 + \cos 2\theta) - 1$$

$$T_3(\sec \theta_m \cos \theta) = \sec^3 \theta_m (\cos 3\theta + 3 \cos \theta) - 3 \sec \theta_m \cos \theta$$

$$T_4(\sec \theta_m \cos \theta) = \sec^4 \theta_m (\cos 4\theta + 4 \cos 2\theta + 3) - 4 \sec^2 \theta_m (\cos 2\theta + 1) + 1$$

- Cautam coeficientii pentru a obtine un polinom Cebîșev

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

$$\Gamma(\theta) = A \cdot e^{-jN\theta} \cdot T_N(\sec \theta_m \cos \theta)$$

ultimul termen:

$$\dots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$$
$$\dots \Gamma_{(N-1)/2} \cdot \cos \theta \quad n \text{ impar}$$

# Transformatoare cu mai multe sectiuni de tip Cebîșev

- $A, \theta \rightarrow 0$ , liniile de lungime 0, dispar

$$\Gamma(0) = \frac{Z_L - Z_0}{Z_L + Z_0} = A \cdot T_N(\sec \theta_m) \quad A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec \theta_m)} \quad \Gamma_m = |A|$$

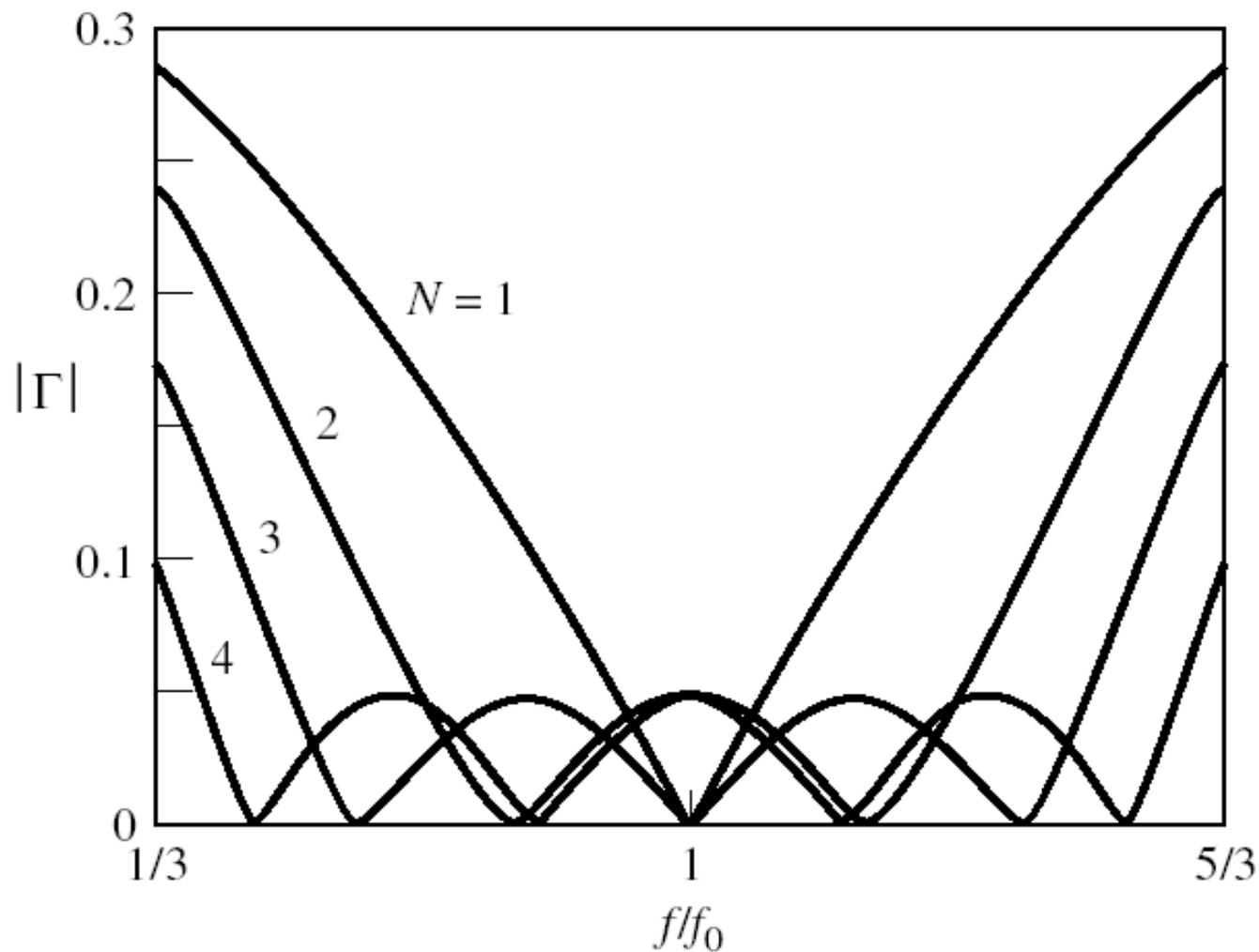
$$T_N(\sec \theta_m) = \frac{1}{\Gamma_m} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \cong \frac{1}{2\Gamma_m} \left| \ln \frac{Z_L}{Z_0} \right|$$

$$T_n(x) = \cosh(n \cosh^{-1}(x))$$

$$\sec \theta_m = \cosh \left[ \frac{1}{N} \cosh^{-1} \left( \frac{1}{\Gamma_m} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \right) \right] \cong \cosh \left[ \frac{1}{N} \cosh^{-1} \left( \left| \frac{\ln(Z_L/Z_0)}{2\Gamma_m} \right| \right) \right]$$

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

# Transformatoare cu mai multe sectiuni de tip Cebîșev



# Transformatoare cu mai multe sectiuni de tip Cebîșev

$Z_L/Z_0$	$N = 2$				$N = 3$					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

$N = 4$

$Z_L/Z_0$	$\Gamma_m = 0.05$				$\Gamma_m = 0.20$			
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

# Exemplu

- Transformator de adaptare cu 3 sectiuni pentru a adapta o sarcina de  $30\Omega$  la o linie de  $100\Omega$  la frecventa  $f_0=3\text{GHz}$ ,  $\Gamma_m=0.1$ 
  - $N = 3$      $Z_L = 30\Omega$      $Z_0 = 100\Omega$

$$\Gamma(\theta) = 2e^{-j3\theta} [\Gamma_0 \cos 3\theta + \Gamma_1 \cos \theta] = Ae^{-j3\theta} T_3(\sec \theta_m \cos \theta)$$

$$|A| = \Gamma_m = 0.1 \quad A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec \theta_m)} \quad Z_L < Z_0 \rightarrow A < 0 \quad A = -0.1$$

$$\sec \theta_m = \cosh \left[ \frac{1}{N} \cdot \cosh^{-1} \left( \left| \frac{\ln Z_L / Z_0}{2\Gamma_m} \right| \right) \right] = \cosh \left[ \frac{1}{3} \cdot \cosh^{-1} \left( \left| \frac{\ln(30/100)}{2 \cdot 0.1} \right| \right) \right] = 1.362$$

$$\theta_m = \arccos \left( \frac{1}{\sec \theta_m} \right) = 0.746 \text{ rad} = 42.76^\circ$$

# Exemplu

$$2[\Gamma_0 \cos 3\theta + \Gamma_1 \cos \theta] = A \sec^3 \theta_m (\cos 3\theta + 3 \cos \theta) - 3A \sec \theta_m \cos \theta$$

$$\cos 3\theta \quad 2\Gamma_0 = A \sec^3 \theta_m \quad \Gamma_0 = -0.1263$$

$$\cos \theta \quad 2\Gamma_1 = 3A(\sec^3 \theta_m - \sec \theta_m) \quad \Gamma_1 = -0.1747$$

simetrie:  $\Gamma_3 = \Gamma_0; \quad \Gamma_2 = \Gamma_1$

# Exemplu

$$n = 0$$

$$\ln Z_1 = \ln Z_0 + 2 \cdot \Gamma_0 = \ln 100 - 2 \cdot 0.1263 = 4.353 \quad \Gamma_0 = -0.1263$$

$$Z_1 = 77.68\Omega \quad \Gamma_1 = -0.1747$$

$$n = 1$$

$$\ln Z_2 = \ln Z_1 + 2 \cdot \Gamma_1 = \ln 77.68 - 2 \cdot 0.1747 = 4.003$$

$$Z_2 = 54.77\Omega$$

$$n = 2$$

$$\ln Z_3 = \ln Z_2 + 2 \cdot \Gamma_2 = \ln 54.77 - 2 \cdot 0.1747 = 3.654$$

$$Z_3 = 38.62\Omega$$

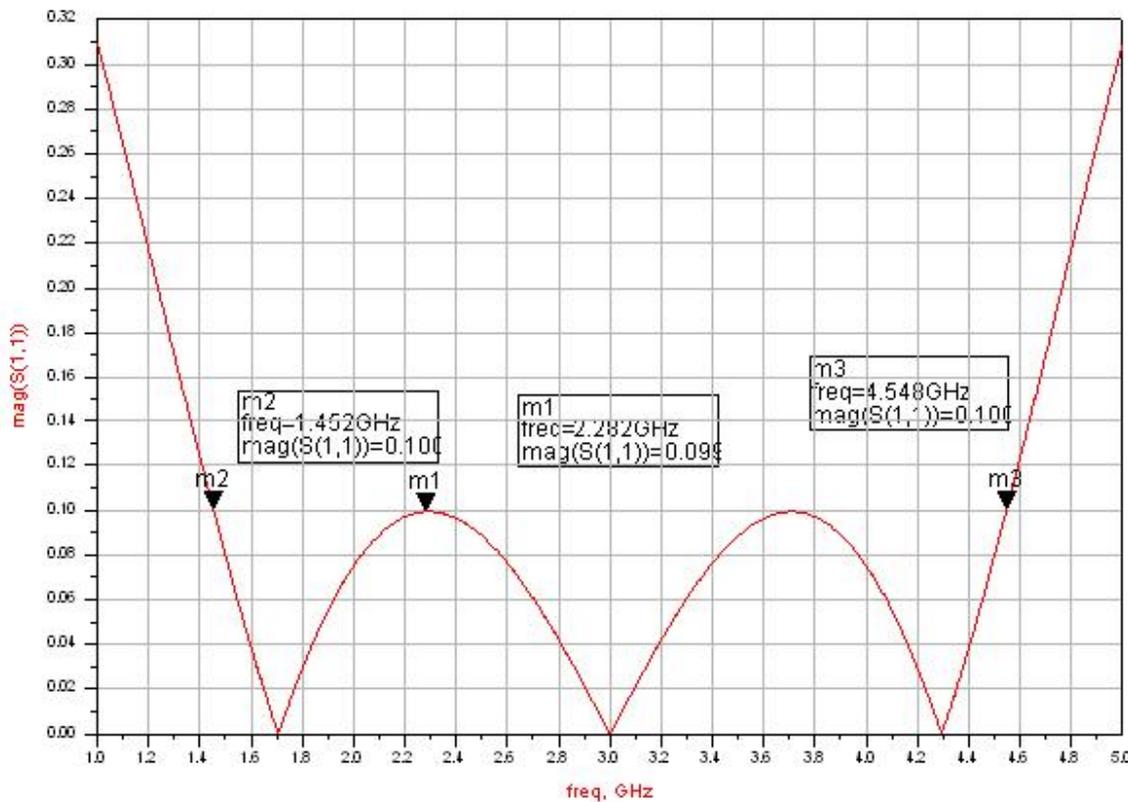
# Exemplu

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4 \cdot 42.76^\circ}{180^\circ} = 1.045$$

$$\Delta f = 3.15 \text{GHz}$$

# Simulare

## ■ Similar Lab. 1



$$\Delta f = 3.096 \text{GHz}$$

$$|\Gamma(3 \text{GHz})| = 4.17 \cdot 10^{-5}$$

$$|\Gamma(2.282 \text{GHz})| = 0.09925$$

# Contact

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